UF Herbert Wertheim College of Engineering
UNIVERSITY of FLORIDA

POWERING THE NEW ENGINEER TO TRANSFORM THE FUTURE

Department of Electrical & Computer Engineering

Summer Robotic Research Experience

Introduction

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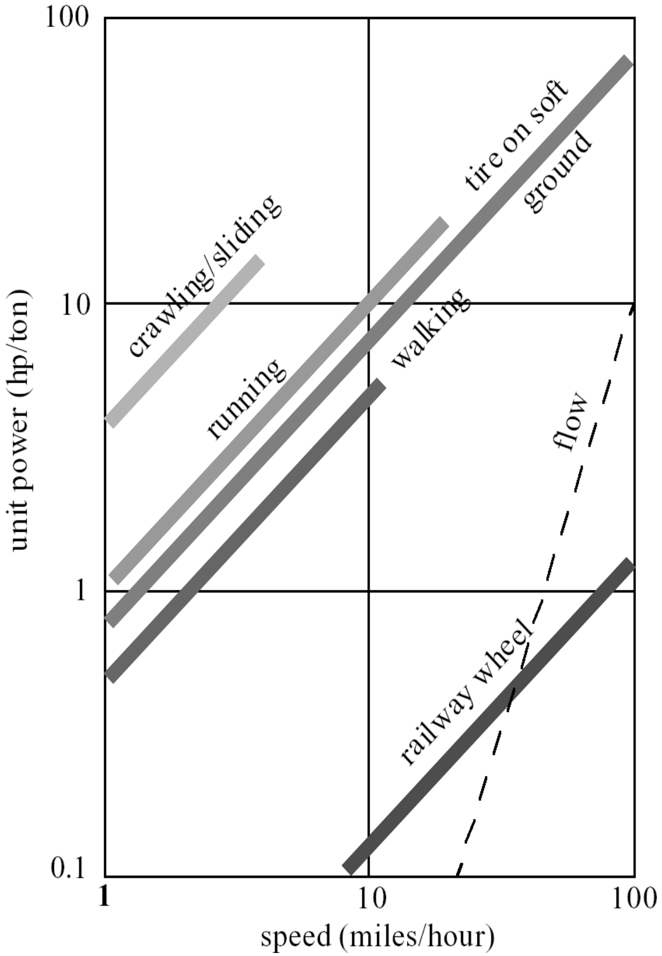
Wheeled Locomotion

Selection of Locomotion Concept



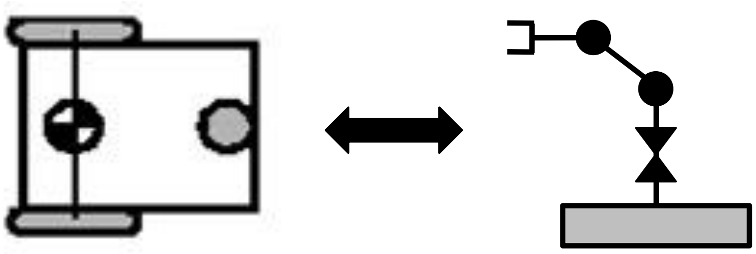
- Wheeled motion is
  - highly efficient on hard and flat surfaces (usually man-made)
- generally restricted to man-made structures
- the de-facto standard for mobile robotics

Review: Dimensionality

- Kinematics
  - Origin: kinein (Greek) – to move
  - The subfield of Mechanics dealing with motions of bodies
- Forward kinematics
  - Given is a set of actuator positions
  - Determine corresponding reference pose
- Inverse kinematics
  - Given is a desired reference pose
  - Determine corresponding actuator positions

Dimensionality

- Wheels
  - Are often subject to motion constraints
  - Often do not allow to compute kinematics directly



- Consequently, for most wheeled robots, actuator positions do not map to unique reference poses
  - There is no direct (i.e., instantaneous) way to measure a robot's position
  - Position must be integrated over time, depends on the path taken
- Understanding mobile robotic motion requires an understanding of wheel constraints placed on the robot's mobility

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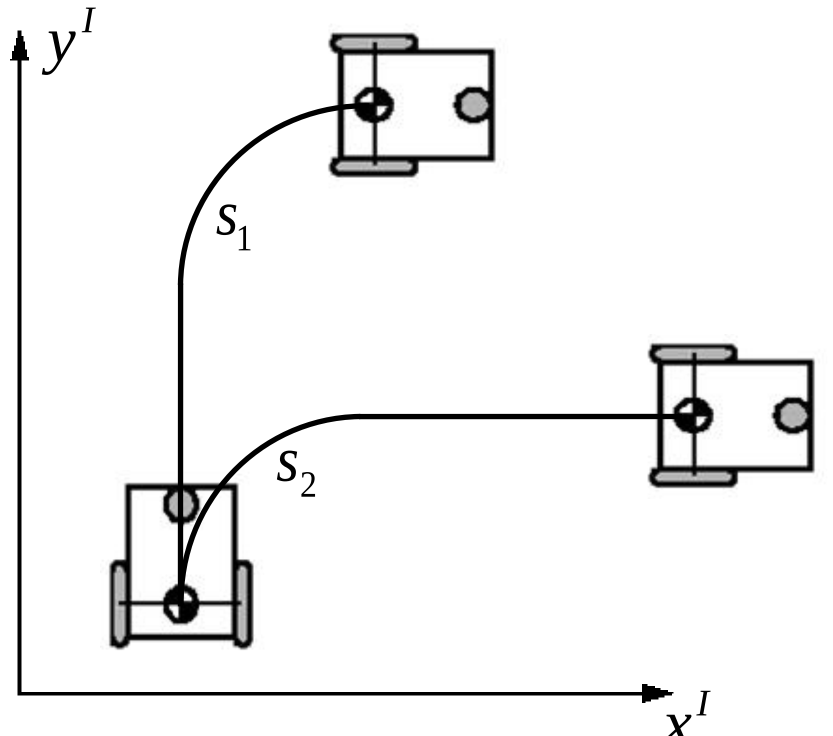
Holonomic Systems

- Holonomic systems
- Diff. eqn. of $\dot{\xi}^I$ are integrable to the final position
- the measure of the traveled distance of each wheel is sufficient to calculate the final position of the robot
- Examples
  - Ballbot
  - Robots composed out of (multiple) wheels that do not constrain motion (i.e., Castor, Swedish and Omni-wheels)

Non-Holonomic Systems

- Non-holonomic systems
  - Diff. eqn. of $\dot{\xi}^I$ are *not integrable* to the final position
  - The measure of the traveled distance *s* of each wheel is not sufficient to calculate the final robot position
  - Knowledge of the movement as a function of time becomes necessary



Homogeneous Transformation

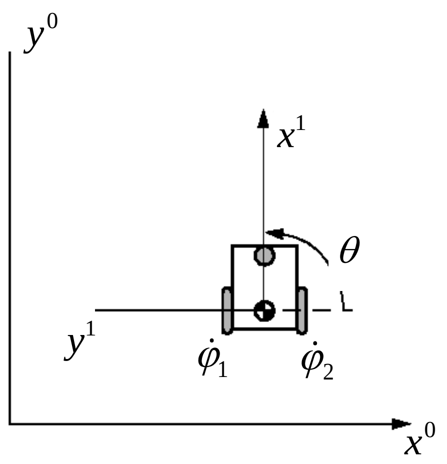
- Representation of differential forward kinematics
  - Robot pose

$$\xi^0 = \left[x_1^0 \; y_1^0 \; \theta_1^0\right]^T$$

  - Mapping velocities between two frames

$$\dot{\xi}^0 = {}^0R^1(\theta)\dot{\xi}^1 = {}^0R^1(\theta)\left[\dot{x}^1 \; \dot{y}^1 \; \dot{\theta}^1\right]^T$$

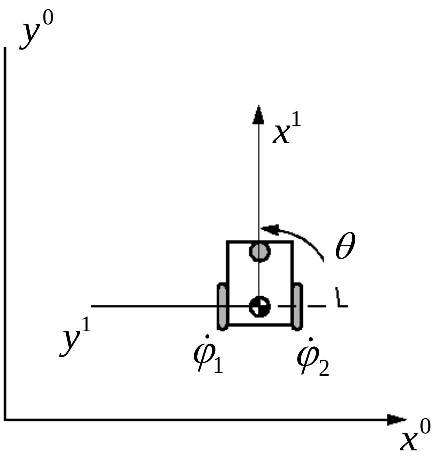
$${}^0R^1(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



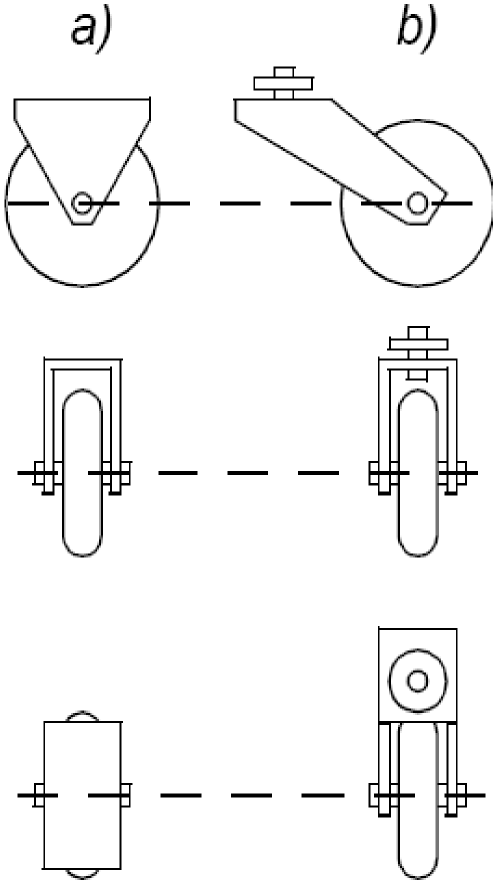
Assumptions

- For robots containing (several) actuated wheels, find $\dot{\xi}^0 = f(\dot{\varphi}_1, \cdots, \dot{\varphi}_n)$ with $\dot{\varphi}_i$ the actuator velocities

- Assumptions
  - Movement on a horizontal plane
  - Point contact of the wheels
  - Wheels not deformable
  - Pure rolling, no slipping, skidding or sliding
  - No friction for rotation around contact point
  - Wheels connected to a rigid frame (chassis)

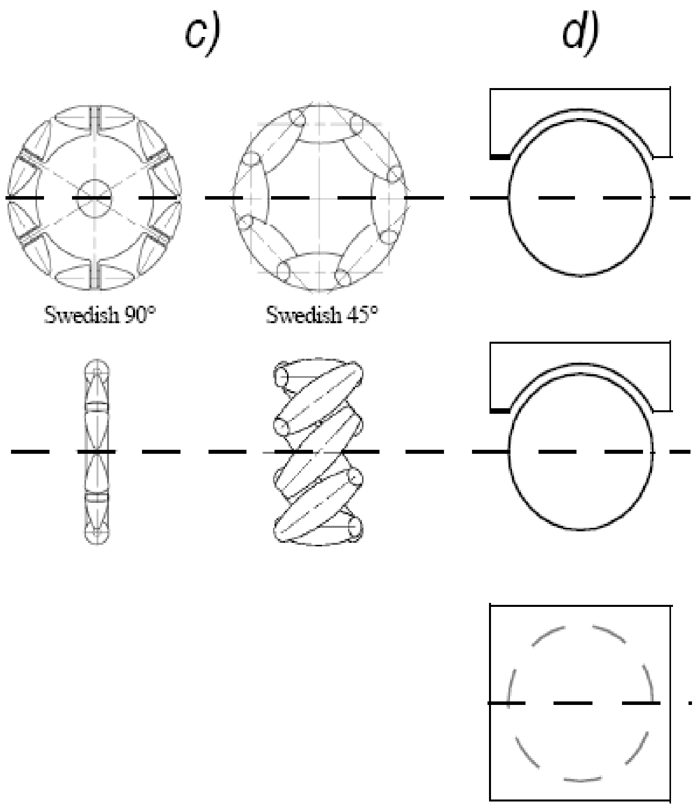


Standard and Castor Wheels



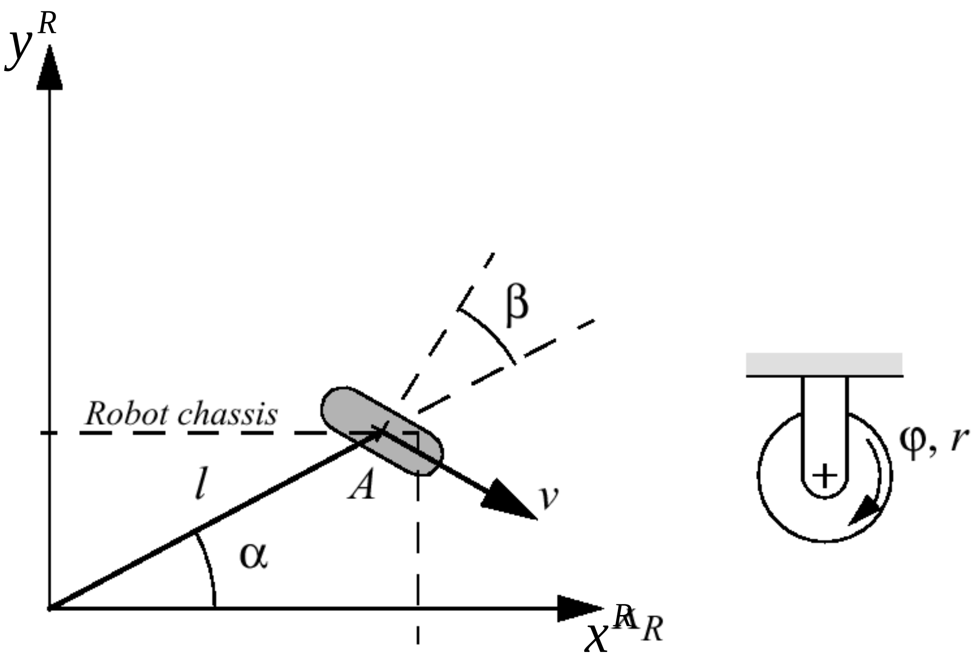
- Standard wheel *a)*
  - two degrees of freedom
  - rotation around the (motorized) wheel axle and the contact point
- Castor wheel *b)*
  - three degrees of freedom
  - rotation around the wheel axle, the contact point and the castor axle

Swedish and Spherical Wheels



- Swedish wheel *c)*
  - three degrees of freedom
  - rotation around the (motorized) wheel axle, around the rollers and around the contact point
- Ball (spherical wheel) *d)*
  - three degrees of freedom
  - suspension technically not solved

Differential Forward Kinematics
Fixed Standard Wheel



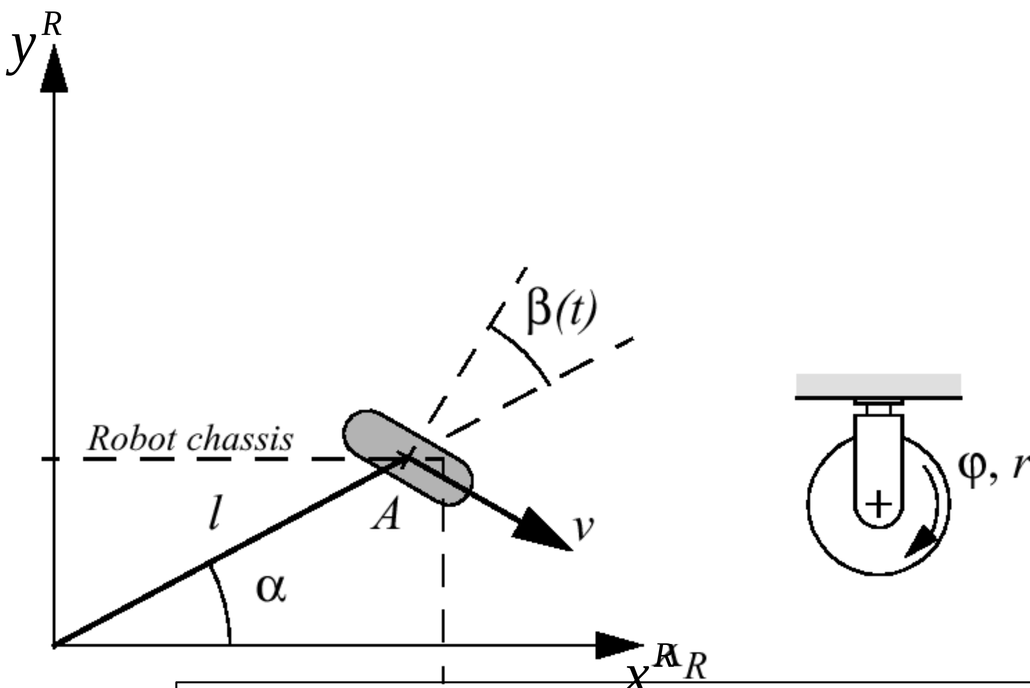
$$\dot{\xi}^R = \left[\dot{x}^1\ \dot{y}^1\ \dot{\theta}^1\right]^T$$

$$\left[\sin(\alpha+\beta)\ \ -\cos(\alpha+\beta)\ \ -l\cos\beta\right] R(\theta)^T \dot{\xi}^I - r\dot{\varphi} = 0$$

$$\left[\cos(\alpha+\beta)\ \ \sin(\alpha+\beta)\ \ l\sin\beta\right] R(\theta)^T \dot{\xi}^I = 0$$

Differential Forward Kinematics

Steered Standard Wheel



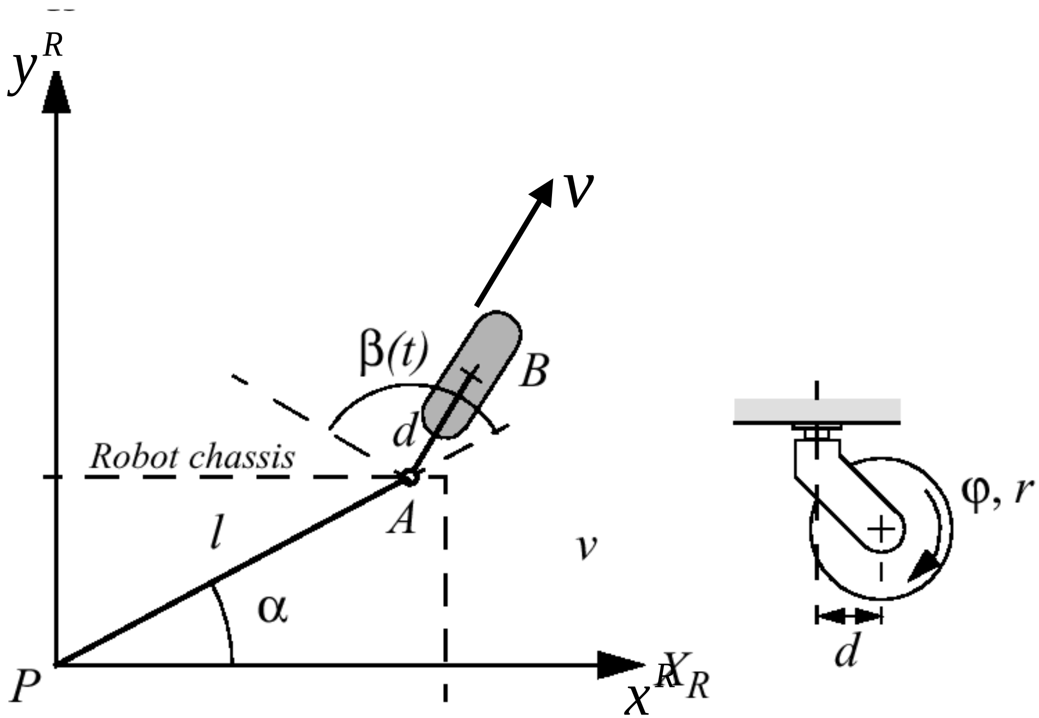
$$\dot{\xi}^R = \left[\dot{x}^1 \; \dot{y}^1 \; \dot{\theta}^1\right]^T$$

$$\left[\sin(\alpha+\beta) \;\; -\cos(\alpha+\beta) \;\; -l\cos\beta\right] \underbrace{R(\theta)^T \dot{\xi}^I}_{\dot{\xi}^R} - r\dot{\varphi} = 0$$

$$\left[\cos(\alpha+\beta) \;\; \sin(\alpha+\beta) \;\; l\sin\beta\right] R(\theta)^T \dot{\xi}^I = 0$$

Differential Forward Kinematics

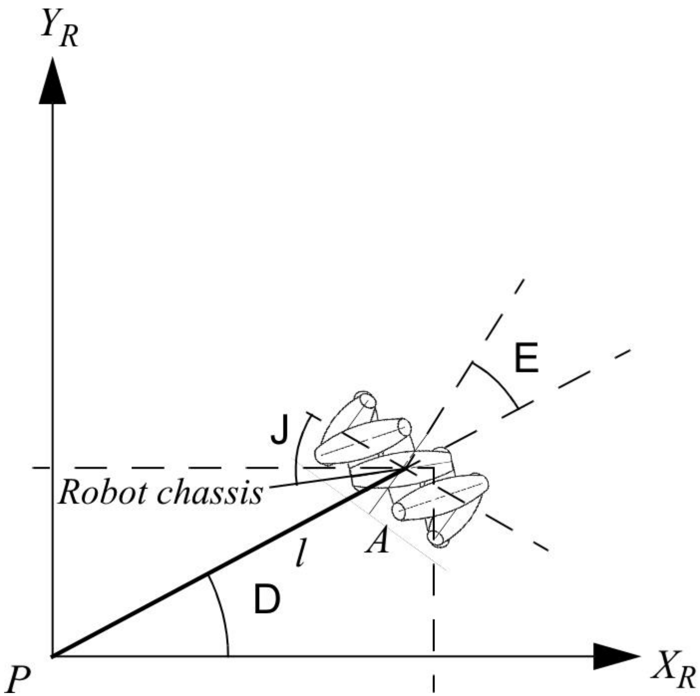
Castor Wheel

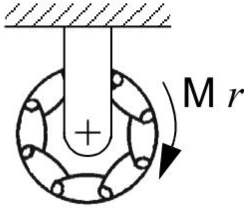


$$\begin{bmatrix}\sin(\alpha+\beta) & -\cos(\alpha+\beta) & -l\cos\beta\end{bmatrix} R(\theta)^T \dot{\xi}^I - r\dot{\varphi} = 0$$

$$\begin{bmatrix}\cos(\alpha+\beta) & \sin(\alpha+\beta) & d + l\sin\beta\end{bmatrix} R(\theta)^T \dot{\xi}^I + d\dot{\beta} = 0$$

Differential Forward Kinematics
Swedish Wheel

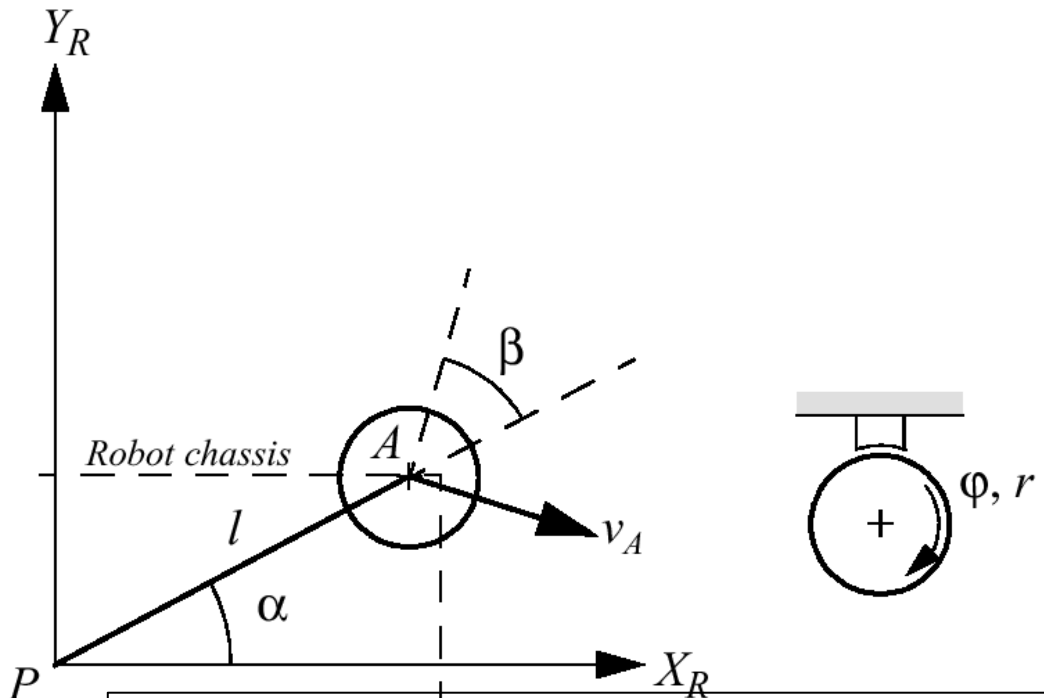




$$\left[\sin(\alpha+\beta+\gamma)\quad -\cos(\alpha+\beta+\gamma)\quad -l\cos(\beta+\gamma)\right]R(\theta)^T\dot{\xi}^I - r\dot{\varphi}\cos\gamma = 0$$

$$\left[\cos(\alpha+\beta+\gamma)\quad \sin(\alpha+\beta+\gamma)\quad l\sin(\beta+\gamma)\right]R(\theta)^T\dot{\xi}^I - r\dot{\varphi}\sin\gamma - r_{sw}\dot{\varphi}_{sw} = 0$$

Differential Forward Kinematics Spherical Wheel



$$[\sin(\alpha+\beta) \quad -\cos(\alpha+\beta) \quad -l\cos\beta]R(\theta)^T\dot{\xi}^I - r\dot{\varphi} = 0$$

$$[\cos(\alpha+\beta) \quad \sin(\alpha+\beta) \quad l\sin\beta]R(\theta)^T\dot{\xi}^I = 0$$

Wheel Arrangements
Static Stability of Overall System

- Given a wheeled robot
  - Each wheel imposes ≥ 0 constraints on its motion
  - Only fixed and steerable standard wheels impose no-sliding constraints

- Suppose the robot has $N_f + N_s$ standard wheels of radius r_i, then the individual wheel constraints can be concatenated in matrix form
  - Rolling constraints

$$J_1(\beta_s)R(\theta)\dot{\xi}^I + J_2\dot{\varphi} = 0 \qquad \varphi(t) = \begin{bmatrix} \varphi_f(t) \\ \varphi_s(t) \end{bmatrix}_{(N_f+N_s)\times 1} \quad J_1(\beta_s) = \begin{bmatrix} J_{1f} \\ J_{1s}(\beta_s) \end{bmatrix}_{(N_f+N_s)\times 3} \quad J_2 = diag(r_1 \cdots r_N)$$

  - No-sliding constraints

$$C_1(\beta_s)R(\theta)\dot{\xi}^I = 0 \qquad C_1(\beta_s) = \begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \end{bmatrix}_{(N_f+N_s)\times 3}$$

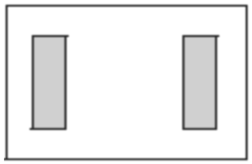
- Solving for $\dot{\xi}^I$ results in an expression for Differential Forward Kinematics

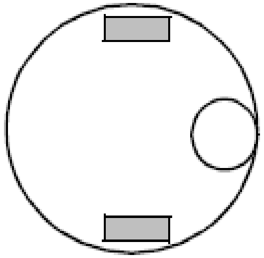
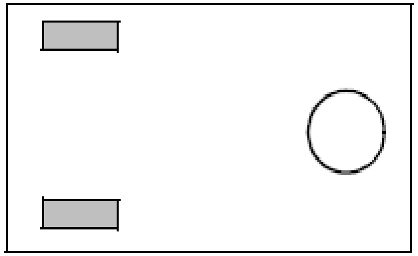
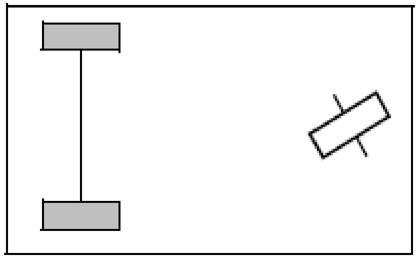
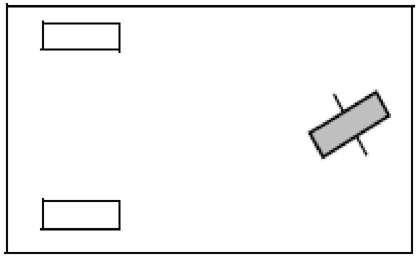
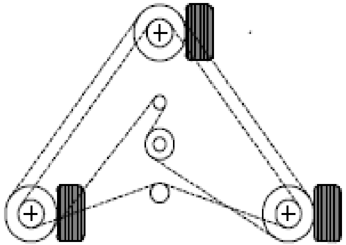
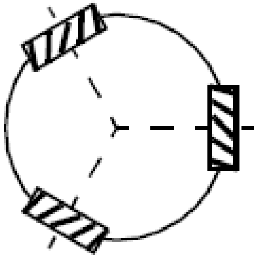
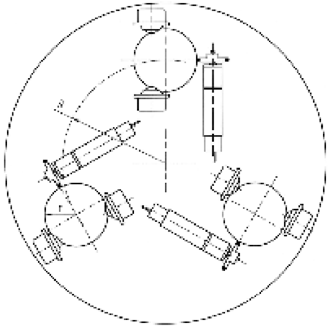
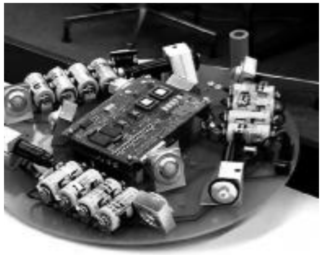
Wheel Arrangements
Static Stability of Overall System

- Stability requires
  - At least 3 wheels in ground contact
  - That CoG lies within support triangle

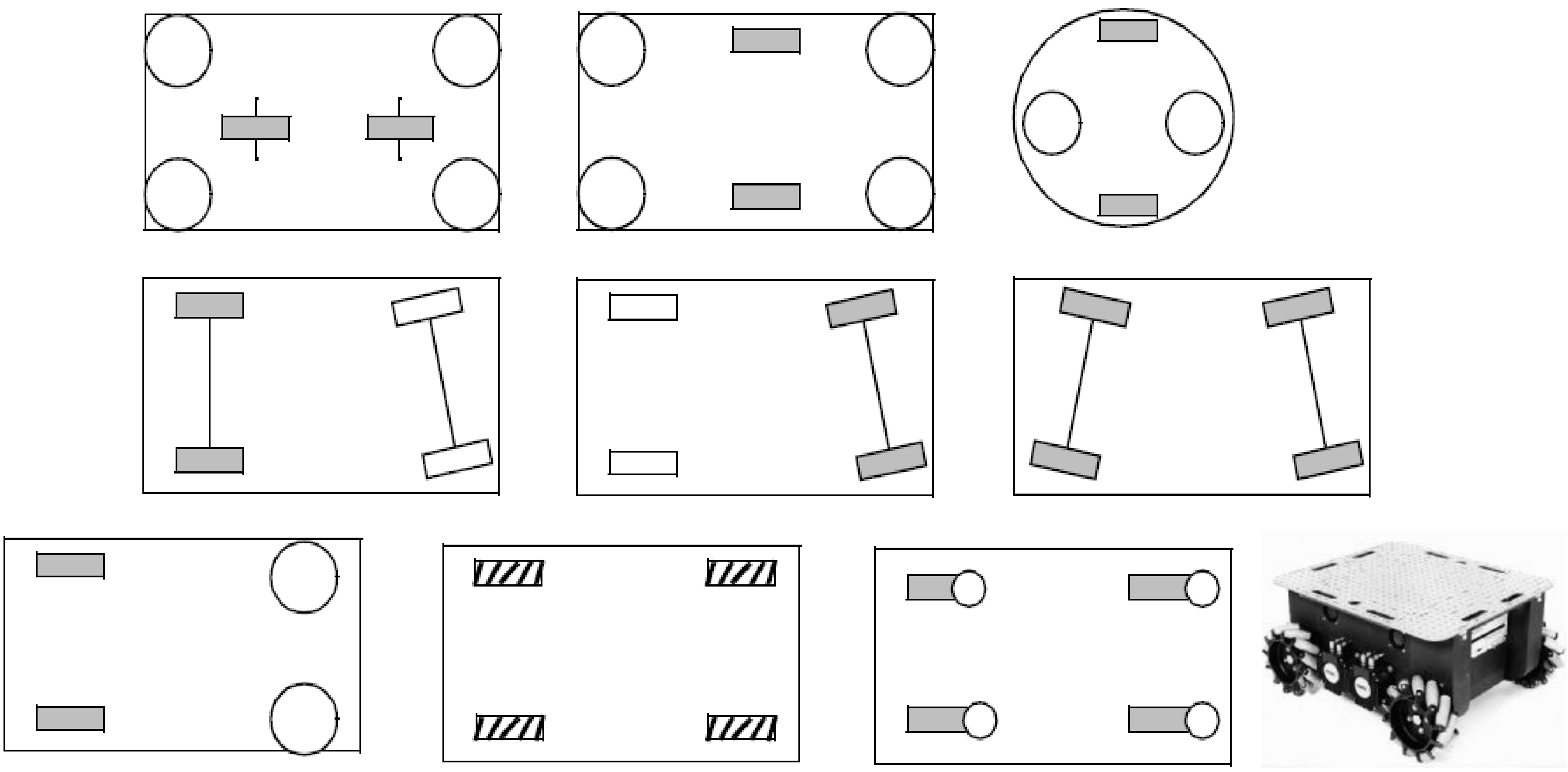
- Stability is improved by 4 and more wheels
  - Such arrangements are hyper static
  - Necessitates a flexible suspension system

Wheel Arrangements
Two and Three Wheeled Robots

Wheel Arrangements
Four Wheeled Robots



Wheel Arrangements
Static Stability of Overall System

- Maneuverabiliy $\delta_M = \delta_m + \delta_s$
  - δ_m degree of mobility
  - δ_s degree of steerability

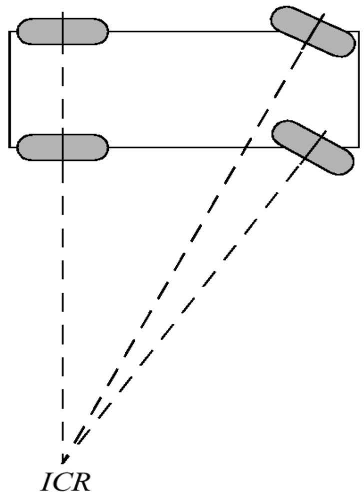
- The maneuverability of a robot is composed of
  - Mobility (restricted by the no-sliding constraints)
  - Additional freedom contributed by the steering mechanisms

Motion Constraints
Instantaneous Center of Rotation

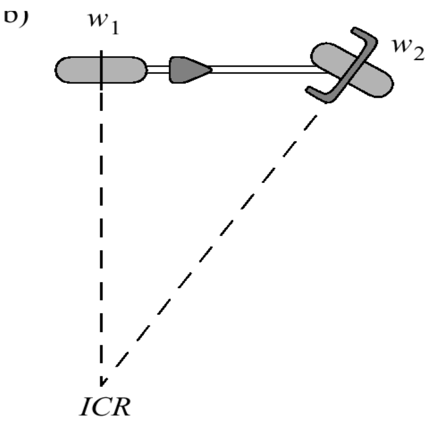
- Instantaneous Center of Rotation (ICR)
  - For any robot with $\delta_M = 2$ the ICR is constrained to lie on a *line*
  - For any robot with $\delta_M = 3$ the ICR can be set to *any point on the 2D plane*

- Examples

Ackermann Steering



Bicycle

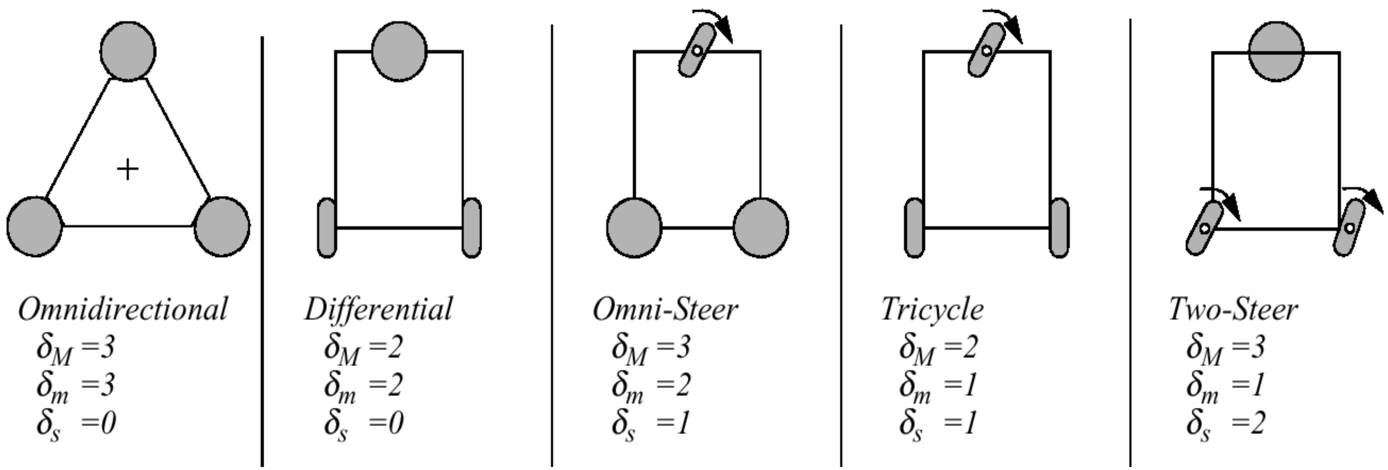


Motion Constraints
Degree of Steerability

- Degree of steerability
  - Represents an *indirect* degree of motion
  - The particular orientation at any instant imposes a kinematic constraint
  - The ability to change that orientation leads to an additional degree of maneuverability
- Range of: $0 \leq \delta_s \leq 2$
- Examples:
  - One steered wheel: Tricycle, $\delta_s = 1$
  - Two steered wheels: Two-steer, $\delta_s = 2$
  - Two steered wheels on a common axis: Car (with Ackermann steering), $\delta_s = 1$

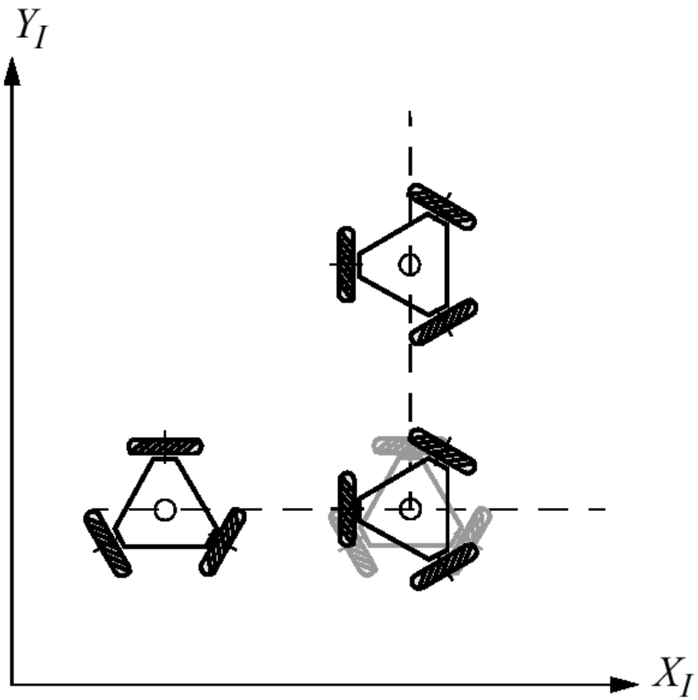
Motion Constraints
Degree of Steerability

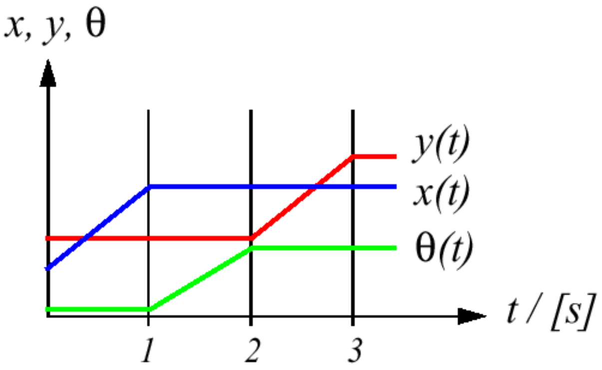
- Degree of Maneuverability $\delta_M = \delta_m + \delta_s$
- Two robots with the same δ_M may not be equivalent instantaneously
- Examples



Motion Constraints
Omni-Drive Example

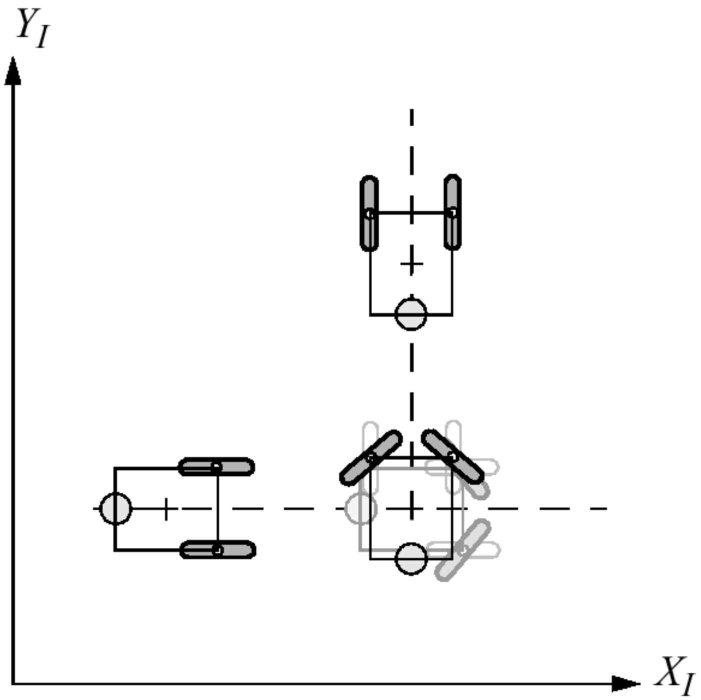
- Omni-Drive

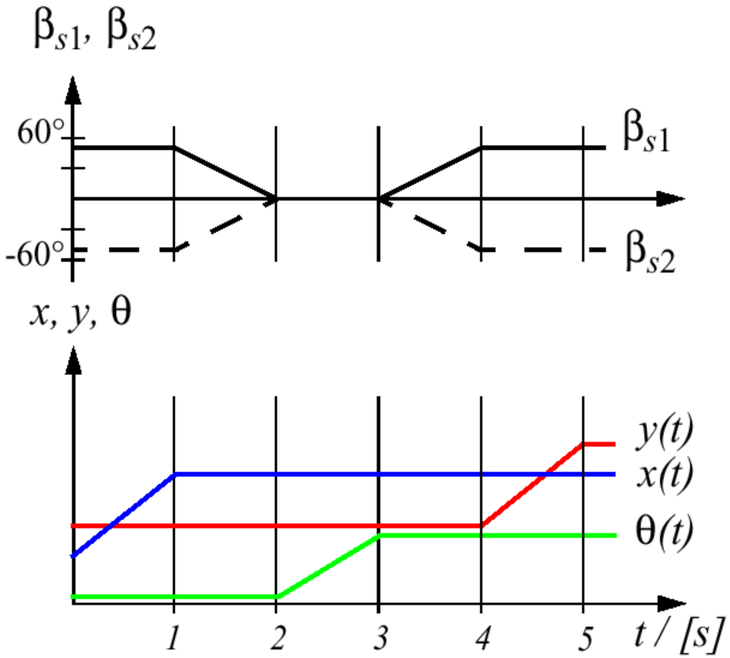




Motion Constraints
Two-Steer Example

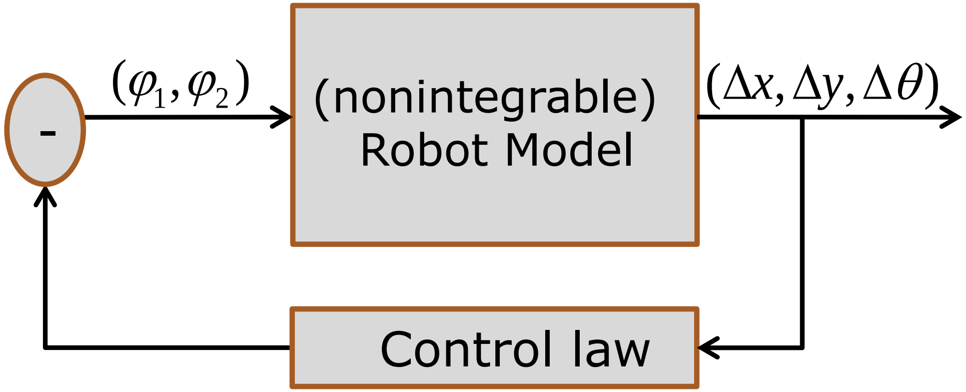
- Two-Steer

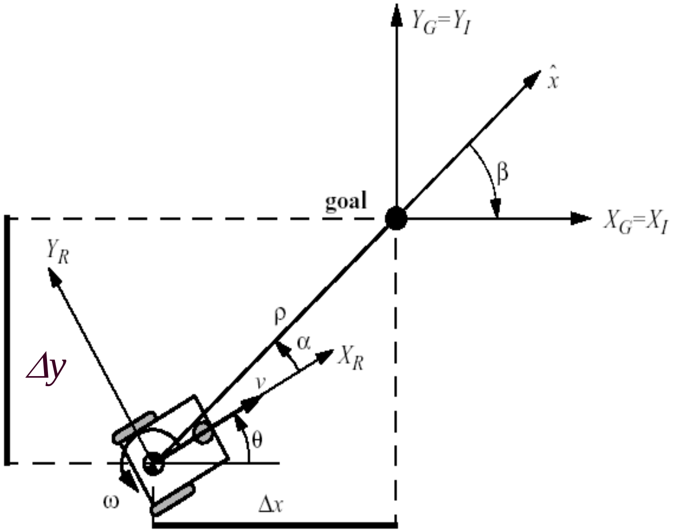




Differential Inverse Kinematics
Iterative Scheme – Control Theory

- In presence of non-holonomic constraints, *diff-erential* inverse kinematics must be considered
  - Transformation between velocities instead of positions
  - Solved via (state) feedback control
- Example: differential drive (kinematic unicycle)





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